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# ENERGY-BASED ESTIMATION OF DAMPING PARAMETERS IN MULTI-DEGREE-OF-FREEDOM VIBRATION SYSTEMS

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## Introduction

This work focuses on balancing applied and dissipated energy to estimate parameters [1,2], in this case for multi-degree-of-freedom (MDOF) and large-order systems.

## Energy Balancing for MDOF Systems: An Example

The equaion of motion of a chain of four unit masses connected by unit linear springs, with linear and nonlinear damping, is $\hat{\mathbf{M}}\ddot{\mathbf{x}} + \hat{\mathbf{C}}\dot{\mathbf{x}} + \hat{\mathbf{K}}\mathbf{x} + \hat{\mathbf{f}}(\dot{\mathbf{x}}, \mathbf{x}) = \hat{\mathbf{r}}(t)$, where $x$ is the displacement vector and $\hat{\mathbf{r}}(t)$ is the input vector, with proportional linear damping, such that $\hat{\mathbf{C}} = c\hat{\mathbf{K}}$, $c$ unknown, and the uniform quadratic damping $d\dot{\mathbf{x}}_i|\dot{\mathbf{x}}_i|$ with unknown constant $d$ is applied to each mass. The excitation $a\cos\omega t$ is applied to the fourth mass. We simulated responses with the values $c$ = 0.15 and $d$ = 0.10, at a step size of $h$ = 0.1 for 1000 steps to remove the transients, and then 200 more steps to record steady state responses. We resonated the first mode ($\omega = 0.618$) with input amplitudes $a$ = 0.25, $a$ = 0.5, $a$ = 0.75, and $a$ = 1. Figure 1 shows the animated response through a cycle of vibration for $a = 1.0$.

The positions are measured during a periodic response. The identification equations are formed by the inner product of the velocity vector with the equation of motion. Integrating over a period of oscillation, the conservative terms drop out, and the nonconservative work terms balance as

$$W_d = \oint \dot{\mathbf{x}}^T \hat{\mathbf{C}}\dot{\mathbf{x}} dt + \oint \dot{\mathbf{x}}^T \hat{\mathbf{f}}(\dot{\mathbf{x}}) dt = \oint \dot{\mathbf{x}}^T \hat{\mathbf{r}}(t) dt = W_a. \quad (1)$$

Equation (1) has embedded unknowns $c$ and $d$, and has the form $c\alpha + d\beta = \gamma$, where $\alpha = \oint (\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2 + \dot{x}_4^2 - \dot{x}_1\dot{x}_2 - \dot{x}_2\dot{x}_3 - \dot{x}_3\dot{x}_4) dt$, $\beta = \oint (|\dot{x}_1|^3 + |\dot{x}_2|^3 + |\dot{x}_3|^3 + |\dot{x}_4|^3) dt$, and $\gamma = \oint a\dot{x}_4 \cos(\omega t) dt$. These integrals were performed using the rectangular integration rule. For each excitation amplitude, we completed this equation, and the least squares solution led to estimates $c_I = 0.1499$ and $d_I = 0.1003$.

If there are more degrees of freedom than sensors, we use proper orthogonal decomposition (POD) for reduced-order modeling. Suppose the "dominant" proper orthogonal modes (POMs) are determined (from the eigenvectors of $\mathbf{R} = \mathbf{X}^T\mathbf{X}/N$, where the rows of $\mathbf{X}$ are the $N$ time samples of elements of $\mathbf{x}$). We write $\mathbf{x} \approx \mathbf{U}\mathbf{y}$ , where columns of $\mathbf{U}$ are the dominant POMs, and $\mathbf{y}$ are the proper orthogonal modal coordinates. If retained modes represent a large percentage of the signal power (for example 99.9%), the approximate equality is rather good. Substituting into the equation of motion and premultiplying by $\mathbf{U}^T$ yields

$$\mathbf{U}^T\hat{\mathbf{M}}\mathbf{U}\ddot{\mathbf{y}} + \mathbf{U}^T\hat{\mathbf{C}}\mathbf{U}\dot{\mathbf{y}} + \mathbf{U}^T\mathbf{K}\hat{\mathbf{U}}\mathbf{y} + \mathbf{U}^T\hat{\mathbf{f}}(\mathbf{U}\mathbf{y}, \mathbf{U}\dot{\mathbf{y}}) \approx \mathbf{U}^T\hat{\mathbf{r}}(t), \quad (2)$$

or $\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} + \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}) \approx \mathbf{r}(t)$. Thus, for the energy balance step, $\oint \dot{\mathbf{y}}^T(\mathbf{C}\dot{\mathbf{y}} + \mathbf{f}(\mathbf{y}, \dot{\mathbf{y}}))dt \approx \oint \dot{\mathbf{y}}^T\mathbf{r}(t)dt$, which is the identification equation of the form $c\alpha + d\beta = \gamma$.

Using the same data, the dominant POM of each response contained 96.9%, 97.5%, 98.1%, and 98.8%, respectively, of the

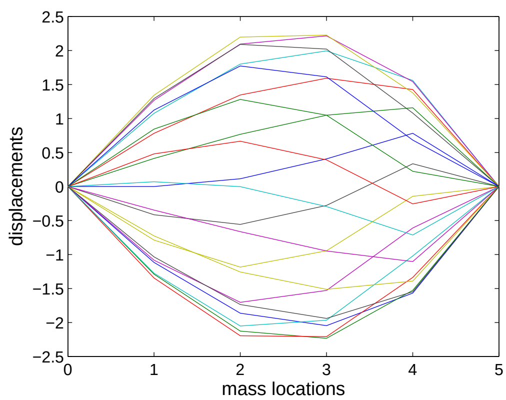


Figure 1. Axial steady-state displacement plotted transversally ($a = 1$).

total signal energy. The identification results were $c_I = 0.1404$ and $d_I = 0.1138$, the error induced by reduced order modeling.

We applied this to a string, robustly to added sensor noise.

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## References

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[2] J.-W. Liang and B. F. Feeny, 2006, *Journal of Sound and Vibration* **295** 988-998.